XXIV.—*On Instruments and Observations for Longitude for Travellers on Land.* By Col. G. Everest, v.p.r.g.s., &c. &c.

Sir,—In a former paper I addressed some remarks to the President and Council of the Royal Geographical Society relative to the subject of observations for longitude by travellers on land; and as a wish was expressed on that occasion that I should revert to the subject, I have taken the earliest opportunity at my leisure to comply therewith.

It is manifest that besides the instability of the basis on which an observer stands at sea, there is this essential difference between the conditions of the seaman and landsman: the former requires the result of his labours for immediate use, whilst to the latter, time is of little or no moment; and provided his data be skilfully acquired and carefully and legibly registered, it is virtually immaterial whether at the instant, or after an interval of months, the result be arrived at. Hence we see that an acquaintance with the principles of spherical trigonometry, or, at any rate, a practical readiness in applying the rules and formulæ of that branch of mathematics, is indispensable to the nautical man, whilst the land-traveller needs no preparatory knowledge whatever in the work of computation.

Now the qualifications which we desire to be possessed by travellers should, like their baggage and other equipments, be reduced to the smallest compass; for geographical data of the highest order are of no intrinsic value, and serve solely to localize the names, scenery, manners, productions, and so forth, of the countries visited and described; and by how much we can lighten the task of obtaining those data, by so much do we leave the traveller at leisure to turn his attention to the latter more popular, interesting, and informatory particulars.

An acquaintance to a certain extent with the heavenly bodies is manifestly necessary to every traveller who purposes to determine the positions of places on the earth's surface by lunar observations: for example, he ought to be able to recognize the nine stars and four planets whose calculated distances from the moon are given in the 'Nautical Almanack,' pp. xiii to xviii of each month. Again, it is quite indispensable that the writing, and more particularly the figures, should be legibly and unmistakeably written in the field-book, for if there exists any uncertainty whether 5 or 3, 6 or 0, 9 or 7, and so forth, be meant, the time of those to whose lot the computation eventually falls, will be more or less taken up in reconciling chimeras.

Moreover, the traveller should have at his disposal a serviceable and portable instrument of which he is thoroughly master.

Now all these desiderata are of easy attainment, for a highly efficient altitude and azimuth instrument of the kind I should recommend is to be had for from twenty to thirty guineas; and any person whose faculties of touch and sight are unimpaired, and who has ordinary intelligence and powers of application, can, in the course of a few days, be able to manipulate it satisfactorily. There is indeed no comparison between the difficulty to a beginner of acquiring skill in the use of an altitude and azimuth instrument, on the one hand, and that of a reflecting instrument and false horizon, on the other, whilst the difference of price between the two sets of apparatus is nearly if not quite insignificant.

A practical acquaintance with the method of determining latitudes is of course indispensable, but even in that case it is desirable that the actual elements should be preserved, because, in working up these, there should be no doubt whatever as to the accuracy of the operation, to which there always is a liability when the results alone are given. With these few provisos I submit to the President and Gentlemen of the Council the accompanying form of Registry for Lunar Observations, which seems to comprise all that is requisite, and I will now, as briefly as I can, offer such explanations as appear needful to make the question intelligible.

I have, for convenience sake, assumed a latitude of 30° N. and longitude 90° E. of Greenwich, but as such place on the globe, though certainly existent, is not one with which I can claim a personal acquaintance, I have assigned to it the name of El Majhūl, which means the unknown.

I suppose that the instrument employed is an altitude and azimuth of 6 or 7 inches diameter to the azimuth circle, furnished with three verniers marked A, B, C, and that it has a vertical circle of 6 inches diameter, admitting of being turned over in altitude, and provided with two verniers D, E. Column No. 1 shows the position of the divided face of the vertical circle, whether to the left or right of the observer. Column No. 2 serves to identify the heavenly body observed. Column No. 3 gives the time by watch, which I suppose to be reliable on for accuracy during the intervals, for it must be remarked that it is not absolute time, but the differences due to those intervals that are needed. Columns Nos. 4, 5, 6, give the readings of the verniers, which being 120°, as near as may be, apart, do not need the insertion of the degrees except in the case of vernier A. Columns 7 and 8 give the readings of the verniers of the vertical circle, which is supposed to be divided for altitudes from 0° to 90° on each quadrant.

It is supposed that the movement in azimuth is always continuous, whether from left to right or *vice versâ*, but it must be

here noted that on farther consideration I see reason to deviate from the plan of observing recommended in my former communication; for whereas I therein say that the moon should be the last object observed in the series, I find in truth that the intervals, which it is on all accounts desirable should be as short as possible, will be lessened by considering the moon as the intermediate object, and reducing the positions of the stars to her time considered as invariable.

One series being complete, with the divided face to the observer's left, I suppose the vertical circle to be turned over in altitude, which will bring the divided face to the observer's right, and increase the readings of vernier A from 0° to 180°; and then by the completion of the second series, in like manner, all the errors of collimation and dislevelment of the transit axis, as is elsewhere abundantly shown, will be self-eliminated, and every sixtieth degree will have come under examination.

The inequality in the pivots of the transit axis is a source of error which can only be got rid of by the greatest care on the part of the instrument-maker; but if the observer is desirous to limit his labours to a single series, his instrument must be provided with a riding-level for adjusting the transit axis, and his vertical circle should be capable of reversing Y for Y, as the best method of obtaining the error of collimation. These, however, do not compensate for the absence of the self-elimination principle by the means of the double series as above recommended, which not only gives greater accuracy, but needs no computation or adjustment.

My reasons for leaving space for a second double series are, that the errors of division to which every instrument is more or less liable, may be further corrected by bringing every thirtieth degree under one or other of the verniers; this would give greater prospect of accuracy, but, as it is not absolutely necessary, it may well be left to the option of the observer.

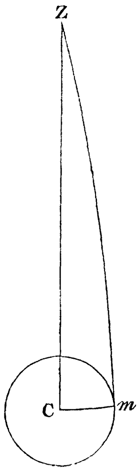
To fill up this form with the observed elements is all that can reasonably be expected from any traveller, and if the President and Gentlemen of the Council approve, I would suggest that a thousand or thereabouts of skeleton forms be lithographed and kept in the Library of the Society, to be supplied as occasion may require to those who are qualified and desirous to employ them in their travels.

I might here close my remarks; but as there doubtless are many travellers who would prefer computing their own observations, and as a desire of this sort meets with my entire sympathy, I hope it will not be deemed intrusive if, on this subject, I add a few words.

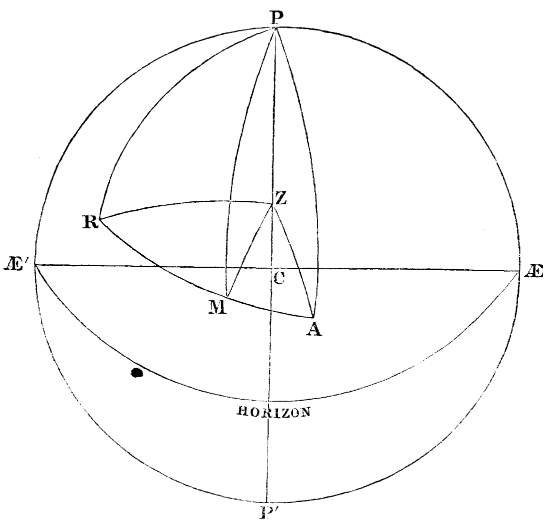
In respect to the well-known corrections in altitude for observed places of the moon, parallax, refraction, semi-diameter, that due to the compression of the earth, &c., it would be quite needless to

say more than that they are well described in Treatises on Navigation, where the requisite Tables are given.

But there is in the case before us one point to be considered, which is not usually referred to in those treatises: I mean the correction in azimuth due to the moon's semi-diameter, which is of this nature. As Z is the pole of the horizon, if two vertical circles be drawn, of which one passes through the centre C of the moon's disk, and the other is tangential thereto, the latter is the one observed, and the former that sought. Now in the triangle Z C m, C m is the moon's semi-diameter, and Z m is the observed zenith distance, or $a_{(1)}$ by my notation; and as this is a right-angled triangle, and C m or r is very small, the value as affects the azimuth is manifestly $Z = r$, sec. a.



In the accompanying diagram, which is intended to represent a stereographic projection of the celestial hemisphere adapted to the occasion of our acquaintance El Majhūl (the eye being at the point where the declination-circle of the nadir meets the equator), R, M, A are the positions of Regulus, the Moon, and Antares. Z is the zenith, P C P′ is the mid-heaven, and Æ C Æ′ is the equator. Now it is evident that the usual ta-



bulated corrections having been applied, so that M may truly represent the moon's centre at the time of observation, the triangle P Z M is constant, and will not be affected by the variations of either R or A.

But in either triangle, P Z R or P Z A, there are only two constants, $P Z = \underset{(1)}{\lambda}$ and P R or $P A = \underset{(1)}{æ}$ or $\underset{(1)}{æ'}$; all the rest being functions of the variable angle P.

Thus $\underset{(1)}{a}$ is a function of P determined by the equation

$$\cos \underset{(1)}{a} = \cos P . \sin \underset{(1)}{\lambda} . \sin \underset{(1)}{æ} + \cos \underset{(1)}{\lambda} . \cos \underset{(1)}{æ},$$

which, because $\underset{(1)}{\lambda}$ and $\underset{(1)}{a}$ are constant, may be put under the form

$$\cos \underset{(1)}{a} = A . \cos P + B,$$

and if $\underset{(1)}{a'}$ be the corrected value and $\underset{(1)}{a'} - \underset{(1)}{a} = \Delta \underset{(1)}{a}$, then

$$\Delta \underset{(1)}{a} = \left\{ \pm \left(\frac{d \underset{(1)}{a}}{d P}\right) . \frac{d P}{1} + \left(\frac{d^2 \underset{(1)}{a}}{d P^2}\right) . \frac{d P^2}{1 . 2} \pm \left(\frac{d^3 \underset{(1)}{a}}{d P^3}\right) . \frac{d P^3}{1 . 2 . 3} \right\} . \operatorname{cosec} 1''.$$

Now, $d P$ is the equivalent of the interval elapsed between the observation of the star and that of the moon; and if n be the number of seconds of time in that interval, then $n \dfrac{3.14159, \&c.,}{12.60.60}$ will be the value of $d P$ in parts of the radius unity; therefore $L (d P) = L (n) + \bar{5}.86167$.

Now, the differential co-efficients are as follows:—

1st. $$\left(\frac{d \underset{(1)}{a}}{d P}\right) = A . \frac{\sin P}{\sin \underset{(1)}{a}}.$$

2nd. $$\left(\frac{d^2 \underset{(1)}{a}}{d P^2}\right) = A \frac{\cos P}{\sin \underset{(1)}{a}} - A^2 . \frac{\sin^2 P . \cos \underset{(1)}{a}}{\sin^3 \underset{(1)}{a}}.$$

3rd. $$\left(\frac{d^3 \underset{(1)}{a}}{d P^3}\right) = - A \frac{\sin P}{\sin \underset{(1)}{a}} - \frac{3}{2} A^2 . \frac{\sin 2 P \cos \underset{(1)}{a}}{\sin^3 \underset{(1)}{a}} + A^3 . \frac{\sin^3 P}{\sin^5 \underset{(1)}{a}} . (2 + \cos 2\underset{(1)}{a}).$$

and the following are the numerical values in the case of the star Regulus, assuming $d P$ to be an increment to the angle P due to an interval of 15 minutes of time:—

$\left(\frac{d \underset{(1)}{a}}{d P}\right) . \frac{d P}{1} . \operatorname{cosec} 1''$	$3^\circ\ 14'\ 33''$
$\left(\frac{d^2 \underset{(1)}{a}}{d P^2}\right) . \frac{d P^2}{1 . 2} . \operatorname{cosec} 1''$	$-\ 10''$
$\left(\frac{d^3 \underset{(1)}{a}}{d P^3}\right) . \frac{d P^3}{1 . 2 . 3} . \operatorname{cosec} 1''$	$-\ 2''$

In a similar manner the correction for the observed azimuth in the like circumstances may be found as thus:—

If Z be the internal angle of the triangle P Z R at the time of

observation, and Z' that corresponding to the time of observing the moon, then, as Z is a function of P,

$$(Z'-Z)=\left\{\pm\left(\frac{dZ}{dP}\right).\frac{dP}{1}+\left(\frac{d^2Z}{dP^2}\right).\frac{dP^2}{1.2}\pm\left(\frac{d^3Z}{dP^3}\right).\frac{dP^3}{1.2.3},\ \&c.\right\}\operatorname{cosec}''.$$

Now the form of this function is derived from the equation

$$\cot Z = \cot \underset{(1)}{\alpha} \,.\, \sin \underset{(1)}{\lambda} \,.\, \operatorname{cosec} P - \cos \underset{(1)}{\lambda} \,.\, \cot P,$$

and differentiating and substituting for $\frac{\sin^2 Z}{\sin^2 P}$ its equal $\frac{\sin^2 \underset{(1)}{\alpha}}{\sin^2 \underset{(1)}{a}}$, there results

$$\left(\frac{dZ}{dP}\right) = \tfrac{1}{2}\sin 2\underset{(1)}{\alpha} \,.\, \sin\underset{(1)}{\lambda} \,.\, \cos P \,.\, \operatorname{cosec}^2 \underset{(1)}{a} - \cos\underset{(1)}{\lambda} \,.\, \sin^2\underset{(1)}{\alpha} \,.\, \operatorname{cosec}^2 \underset{(1)}{a}$$

which, as $\underset{(1)}{\alpha}$ and $\underset{(1)}{\lambda}$ are constant, may be put under the form

$$\left(\frac{dZ}{dP}\right) = A' \,.\, \cos P \,.\, \operatorname{cosec}^2 \underset{(1)}{a} - B' \,.\, \operatorname{cosec}^2 \underset{(1)}{a}.$$

If in this expansion there be put $\left(\frac{d\underset{(1)}{a}}{dP}\right) = X$; $\left(\frac{d^2\underset{(1)}{a}}{dP^2}\right) = Y$, there results

$$\left(\frac{d^2Z}{dP^2}\right) = -A' \,.\, \sin P \,.\, \operatorname{cosec}^2 \underset{(1)}{a} + 2\cot \underset{(1)}{a} \,.\, \operatorname{cosec}^2 \underset{(1)}{a}\, X\,(B' - A' \,.\, \cos P),$$

and, by a continuation of the same process, the differential coefficient $\left(\frac{d^3Z}{dP^3}\right)$ is to be found; but as it is very long, I will spare the President and Gentlemen of the Council further infliction of this dry matter, and confine myself to the computed numerical values, which are as follow :—

$\left(\frac{dZ}{dP}\right).\frac{dP}{1}\operatorname{cosec} 1''$	$-\ 1^\circ\ 48'\ 29''$
$\left(\frac{d^2Z}{dP^2}\right).\frac{dP^2}{1.2}\operatorname{cosec} 1''$	$+\ 47''$
$\left(\frac{d^3Z}{dP^3}\right).\frac{d.P^3}{1.2.3}\operatorname{cosec} 1''$	$\left\{\begin{array}{r} +\ 2'' \\ -\ 10'' \\ -\ 0'' \end{array}\right.$

From these data a judgment may be formed as to how far it may be necessary to carry these computations. It does not follow that the second term of the series for $(\underset{(1)}{a'} - \underset{(1)}{a})$ will always be so small as 10″; neither will the corresponding term of the series for $(Z' - Z)$ always be so large as 47″. One thing only is quite clear, that the smaller the interval due to dP, the lighter also will be the labour of computation.

I will remark, however, that though it is indispensable to compute

the correct value of the term $(\underset{(1)}{a'} - \underset{(1)}{a})$ by the differential process above given, yet that labour may be spared in finding the corresponding value $(Z' - Z)$ by the method which I pointed out in my former communication—as thus :—

$$\sin Z = \frac{\sin P}{\sin \underset{(1)}{a}} \cdot \sin \underset{(1)}{x}, \text{ and } \sin Z = \frac{\sin P'}{\sin \underset{(1)}{a'}} \cdot \sin \underset{(1)}{x}$$

whereby the difference $(Z' - Z)$ will be found without differentiation.

A few closing remarks may now be not out of place.

1st. The value of the angle P, due to the instant of observation of the star, must be found by means of the triangle P Z R (or P Z A), whose three sides are given, the method of accomplishing which is so well known as to need no explanation. If this angle be applied to the right ascension of the star, it will give the right ascension of the mid-heaven at that instant.

2nd. The elapsed interval d P, reduced to seconds of a great circle, and applied to the right ascension of the mid-heaven, will give the right ascension of the mid-heaven at the time of observing the moon, or at the value P′ of the angle at P.

3rd. The difference $(\underset{(1)}{a'} - \underset{(1)}{a})$ applied to $\underset{(1)}{a}$ will give the zenith-distance of the star at the time of the observation of the moon.

4th. The difference $(Z' - Z)$ applied to the observed difference of recorded azimuthal readings between the moon and star will give the value of Z in the triangle Z M R (or Z M A).

5th. In the triangle Z M R (or Z M A) thus corrected, the two sides Z M, Z R (or Z A), and the included angle Z, are known, and the arcs M R or (M A) may be determined by any of the known formulæ, the simplest (when a table of natural sines and cosines is at hand) being in my opinion

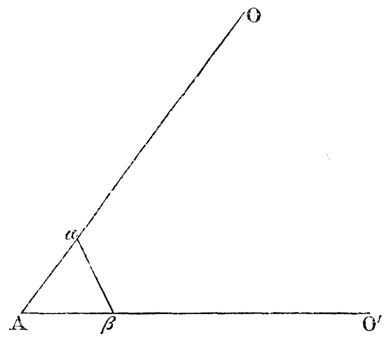
$$\cos \text{M R or } \cos \text{M A} = \cos Z \cdot \sin \text{Z M} \cdot \left\{ \begin{matrix} \sin \text{Z R} \\ (\text{or } \sin \text{Z A}) \end{matrix} \right\} + \cos \text{Z M} \cdot \left\{ \begin{matrix} \cos \text{Z R} \\ \text{or } \cos \text{Z A} \end{matrix} \right\}$$

As to the rest of the operation, it is so familiar that any farther remarks would be superfluous.

Before closing this communication I may as well advert to another subject to which I drew the attention of the President and Gentlemen of the Council some months ago. I mean the measurement of the breadths of rivers or other inaccessible distances by travellers who are not supplied with any instrument for taking angles, in reference to which I have drawn up a table (see p. 323) showing the values of chords, sines, and cosines, of all angles from 30° to 120° inclusive, of the use and application of which I shall be happy to give a detailed explanation if desired.

As travellers are occasionally liable to be without instruments at hand wherewith to measure angles, and may be desirous to measure the breadth of a river or other inaccessible distance, the following method may perhaps be of service :—

How to Measure an Angle, and obtain its Sine and Cosine.

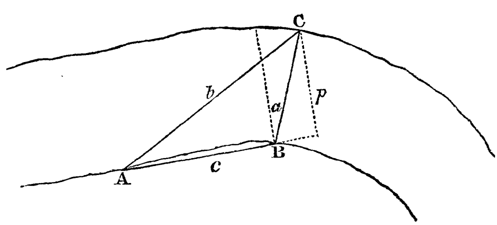


A is the position of the eye ; O, O′ are two distant objects, between which the angle at A is required.

Drive two pegs into the ground, one at a in the direction A O, and another at β in the direction A O′, both at equal distances from A. Measure the distance $a\beta$, and $\frac{a\beta}{Aa}$ will be the chord of the angle A to radius unity, which call γ.

$$\text{Then } \sin A = \gamma \,.\, (1 - \tfrac{1}{4}\gamma^2)^{\frac{1}{2}} \text{ and cosine } A = 1 - \tfrac{1}{2}\gamma^2.$$

If $\frac{1}{2}\gamma^2$ is greater than 1, the angle A will be obtuse and its cosine negative.



Suppose you want to measure the breadth of a river, and that C is an eminent point which can be seen only from two points A and B, between which the distance A B can be conveniently measured. Let A B $= c$, and the perpendicular breadth of the river be denoted by p.

$$\text{Then } p = c \,.\, \frac{\sin A \,.\, \sin B}{\sin A \,.\, \cos B + \cos A \,.\, \sin B}.$$

In this case p is assumed as the perpendicular breadth of a river, and equal to a line parallel to it drawn from A to the opposite bank, which is quite near

enough for rough purposes. If the angle B be nearly a right angle, then B C may be taken as the breadth, and the expression will be

$$a = c \frac{\sin A}{\sin A \,.\, \cos B + \cos A \,.\, \sin B}.$$

If the traveller has a book of pocket logarithms, of course the matter is much simplified, for then $L (\sin \frac{1}{2} A) = L (a\,\beta) + \text{ar. co. } L (A\,a) + \text{ar. co. } L (2)$; and knowing thus $\frac{1}{2}$ A, the value of A is easily obtained. So likewise of B, and the denominator in the above case is only sin (A + B) or its supplement.

The following table may serve in such cases to facilitate computation. Column No. 2 contains the value of the chord to radius unity, or $\frac{a\,\beta}{A\,a}$ above adverted to. Column No. 1 the degree of a circle corresponding thereto. Columns Nos. 3 and 4 the natural sine and cosine of each angle between 30° and 120° inclusive:—

TABLE of ARCS from 30° to 120° inclusive, showing the Natural Chords, Sines, and Cosines of each.

Degrees.	Chords.	Sines.	Cosines.	Degrees.	Chords.	Sines.	Cosines.	Degrees.	Chords.	Sines.	Cosines.
30	·51764	·50000	·86603	60	1·00000	·86603	·50000	90	1·41421	1·00000	·00000
1	·53448	·51504	·85717	1	1·01508	·87462	·48431	1	1·42650	·99985	·01745
2	·55127	·52992	·84805	2	1·03008	·88295	·46947	2	1·43868	·99939	·03490
3	·56803	·54464	·83867	3	1·04500	·89101	·45399	3	1·45705	·99983	·05234
4	·58474	·55919	·82904	4	1·05984	·89879	·43837	4	1·46271	·99756	·06976
5	·60141	·57358	·81915	5	1·07460	·90631	·42262	5	1·47455	·99619	·08716
6	·61803	·58779	·80902	6	1·08928	·91355	·40674	6	1·48629	·99452	·10453
7	·63461	·60182	·79864	7	1·10387	·92050	·39073	7	1·49791	·99255	·12187
8	·65114	·61566	·78801	8	1·11839	·92718	·37461	8	1·50942	·99027	·13917
9	·66761	·62932	·77715	9	1·13281	·93358	·35837	9	1·52081	·98769	·15643
40	·68404	·64279	·76604	70	1·14715	·93969	·34202	100	1·53209	·98481	·17365
1	·70041	·65606	·75471	1	1·16141	·94552	·32557	1	1·54325	·98163	·19081
2	·71674	·66913	·74314	2	1·17557	·95106	·30902	2	1·55429	·97815	·20791
3	·73300	·68200	·73135	3	1·18964	·95630	·29237	3	1·56522	·97437	·22495
4	·74921	·69466	·71934	4	1·20363	·96126	·27564	4	1·57602	·97030	·24192
5	·76537	·70711	·70711	5	1·21752	·96593	·25882	5	1·58671	·96593	·25882
6	·78146	·71934	·69466	6	1·23132	·97030	·25192	6	1·59727	·96126	·27564
7	·79750	·73135	·68200	7	1·24503	·97437	·22495	7	1·60771	·95630	·29237
8	·81347	·74314	·66913	8	1·25864	·97815	·20791	8	1·61803	·95106	·30902
9	·82939	·75471	·65606	9	1·27216	·98163	·19081	9	1·62823	·94552	·32557
50	·84524	·76604	·64279	80	1·28558	·98481	·17365	110	1·63830	·93969	·34202
1	·86102	·77715	·62932	1	1·29890	·98769	·15643	1	1·64825	·93358	·35837
2	·87674	·78801	·61566	2	1·31212	·99027	·13917	2	1·65808	·92718	·37461
3	·89240	·79864	·60182	3	1·32524	·99255	·12187	3	1·66777	·92050	·39073
4	·90798	·80902	·58779	4	1·33826	·99452	.10453	4	1.67734	.91355	.40674
5	.92350	.81915	.57358	5	1·35118	·99619	·08716	5	1·68678	·90631	·42262
6	·93894	.82904	·55919	6	1·36400	·99756	·06976	6	1·69610	·89879	·43837
7	·95432	·83867	·54464	7	1·37671	·99863	·05234	7	1·70528	·89101	·45399
8	·96962	·84805	·52992	8	1·38932	·99939	·03490	8	1·71433	·88295	·46947
9	·98485	.85717	·51504	9	1·40182	·99985	·01745	9	1·72326	·87462	·48431
60	1·00000	·86603	·50000	90	1·41421	1·00000	·00000	120	1·73205	·86603	·50000

OBSERVATIONS

OBSERVATIONS for LUNAR DISTANCE taken at El Majhūl in Thibet between Noon and Midnight at 9 P.M. of June 20, 1858. Approximate Latitude 30° North, Longitude 90° East. Watch used 0m. 0s. slow of {M. S. time. / Sidereal time.} Rate {losing / gaining} 0m. 0s. per day.

Face of the Vertical Circle.	Names of Objects observed.	Times by Watch.			Readings of Azimuth Circle.								Readings of Vertical Circle.					Remarks.
					A.			B.		C.			D.			E.		
Left	Regulus . . .	8	45	0	91	21	15	21	10	21	15		23	39	35	39	35	All three objects to the south of the zenith. Regulus and the moon to the west of the meridian. Antares to the east of the meridian. The West and Upper Limb of the moon were observed.
Left	☽'s W. & U. Limb	8	50	0	26	16	40	16	35	16	40		42	46	40	46	40	
Left	Antares . . .	8	55	0	336	31	10	31	0	31	5		29	52	5	52	0	
Right	Antares . . .	9	5	0	158	56	15	56	10	56	10		30	41	15	41	10	
Right	☽'s W. & U. Limb	9	10	0	211	58	20	58	25	58	15		40	40	15	40	15	
Right	Regulus . . .	9	15	0	274	58	35	58	35	58	35		17	11	30	11	25	
Right	Regulus . . .																	
Right	☽'s W. & U. Limb																	
Right	Antares . . .																	
Left	Antares . . .																	
Left	☽'s W. & U. Limb																	
Left	Regulus . . .																	

10, *Westbourne Street, Hyde Park, W.*
November 7th, 1859.